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Technical Notes

Scaling of Transient Particle-Fluid Heat Transfer in Brownian Motion

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Nomenclature

A	=	area
c_p	=	specific heat
d	=	diameter
h	=	convective heat transfer coefficient
k	=	thermal conductivity
k_b	=	Boltzmann constant
Nu	=	Nusselt number
Pr	=	Prandtl number
Re	=	Reynolds number
T	=	temperature
t	=	time
$\bar{u}$	=	average particle velocity
V	=	volume
μ	=	viscosity
Π	=	nondimensional parameter
ρ	=	density
ϕ	=	volume fraction

Subscripts

d	=	diameter
f	=	fluid
i	=	initial
p	=	particle

Superscript

$*$	=	nondimensional

I. Introduction

NANOFLUIDS, or fluids with nanoscale particles, offer enhanced heat transfer compared with other fluids [1–3]. The effective thermal conductivity appears to increase with temperature and particle concentration, and decrease with particle size. There is considerable controversy over the mechanisms involved, with enhanced particle-fluid heat transfer [4,5], Brownian motion [6–10], and particle interactions at the boundary [11] held up as potential explanations. The present work models the transient heat transfer between nanoparticles and the surrounding fluid, providing an estimate of the time scale required to reach equilibrium.

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II. Mathematical Formulation

If nanoparticles with an initial temperature $T_{p,i}$ are introduced into a fluid with an initial temperature $T_{p,f}$, the particles and the fluid will exchange energy until they reach equilibrium. The rate of change of the particle temperature will be given by

$$\rho_p c_{p,p} V_p \frac{\partial T_p}{\partial t} = A_p h (T_f - T_p) \tag{1}$$

where V_p is the volume of the particle, A_p is the surface area of the particle, ρ_p is the particle density, $c_{p,p}$ is the particle specific heat, and h is the convective heat transfer coefficient. For this analysis, the particles are assumed to be spheres with a diameter d_p.

The nondimensional heat transfer coefficient, or Nusselt number of the particle, is defined as

$$Nu_d = d_p h / k_f \tag{2}$$

where h is the convective heat transfer coefficient, and k_f is the thermal conductivity of the fluid.

For a sphere in laminar flow, the Nusselt number will be a function of the Reynolds number Re_d and the fluid Prandtl number Pr_f [12]:

$$Nu_d = 2.0 + 0.6 Re_d^{1/2} Pr_f^{1/3} \tag{3}$$

The Reynolds number is defined as

$$Re_d = \rho_f d_p \bar{u} / \mu_f \tag{4}$$

where $\bar{u}$ is the average particle velocity, ρ_f is the fluid density, and μ_f is the fluid viscosity.

The scaling of this equation is complicated by the fact that velocity of the particles will depend upon the particle temperature [13]. The velocity of the particles due to Brownian motion will be a function of the particle temperature, the Boltzmann constant k_b, the particle density, and the particle diameter [14]:

$$\bar{u} = \sqrt{\frac{18 k_b T_p}{\pi \rho_p d_p^3}} \tag{5}$$

If Eqs. (2–5) are substituted into Eq. (1), the following expression is obtained:

$$\frac{\partial T_p}{\partial t} = \frac{12 k_f}{\rho_p c_{p,p} d_p^2} \left[1.0 + 0.3 \left(\frac{\rho_f}{\mu_f} \right)^{1/2} \left(\frac{18 k_b T_p}{\pi \rho_p d_p} \right)^{1/4} Pr_f^{1/3} \right] [T_f - T_p] \tag{6}$$

This equation can be simplified by using a nondimensional time t^*, nondimensional temperatures T^*, and a nondimensional group Π_1:

$$t^* = \left(\frac{12 k_f}{\rho_p c_{p,p} d_p^2} \right) t = \frac{t}{t_c}, \qquad t_c = \left(\frac{\rho_p c_{p,p} d_p^2}{12 k_f} \right) \tag{7}$$

$$T_f^* = T_f / T_{p.i} \tag{8}$$

$$T_p^* = T_p / T_{p.i} \tag{9}$$

$$\Pi_1 = 0.3 \left(\frac{\rho_f}{\mu_f} \right)^{1/2} \left(\frac{18 k_b T_{p,i}}{\pi \rho_p d_p} \right)^{1/4} Pr_f^{1/3} \tag{10}$$

The simplified equation heat transfer equation will be

$$\frac{\partial T_p^*}{\partial t^*} = [1.0 + \Pi_1 (T_p^*)^{1/4}][T_f^* - T_p^*] \tag{11}$$

Figure 1 shows representative values of t_c as a function of particle diameter for aluminum and copper particles in water and ethylene glycol, using the material properties at 300 K [12].

The heat transfer equation can be simplified using the first law of thermodynamics. If there is no external heat addition, the energy equation is

$$(1-\phi)\rho_f c_{p,f} T_{f,i} + \phi \rho_p c_{p,p} T_{p,i} = (1-\phi)\rho_f c_{p,f} T_f + \phi \rho_p c_{p,p} T_p \tag{12}$$

where $c_{p,f}$ is the fluid specific heat, and ϕ is the volume concentration of the particle.

Dividing through by $T_{p,i}$ and rearranging the expression, yields

$$\begin{aligned} T_f^* &= T_{f,i}^* + \left[\left(\frac{\phi}{1-\phi}\right)\left(\frac{\rho_p}{\rho_f}\right)\left(\frac{c_{p,p}}{c_{p,f}}\right)\right](1 - T_p^*) \\ &= \Pi_2 + \Pi_3 (1 - T_p^*) \end{aligned} \tag{13}$$

where

$$\Pi_2 = \left(\frac{\phi}{1-\phi}\right)\left(\frac{\rho_p}{\rho_f}\right)\left(\frac{c_{p,p}}{c_{p,f}}\right) \tag{14}$$

$$\Pi_3 = T_{f,i}^* \tag{15}$$

Substituting this expression into (11) yields an ordinary differential equation for the system:

$$\frac{\partial T_p^*}{\partial t^*} = [1.0 + \Pi_1 (T_p^*)^{1/4}][(\Pi_2 + \Pi_3) - (1 + \Pi_2) T_p^*] \tag{16}$$

The initial condition of this equation is

$$T_p^*(t^* = 0) = 1 \tag{17}$$

III. Numeric Solution

This equation can be solved numerically with a simple marching scheme. Figure 2 shows T_p^* as a function of t^* for aluminum nanoparticles in water, with an initial temperature for the particles of 320 K, and initial temperature for the water of 300 K, and a volume fraction of 0.01. This corresponds to changing Π_1 while keeping Π_2 and Π_3 constant. These results show that as the particle diameter becomes greater than 500 nm, the size effects disappear in the transient solution. Because Π corresponds to the Reynolds-number dependent portion of the heat transfer correlation, this also shows that the difference in fluid-particle heat transfer due to particles accelerating and decelerating has a quantifiable effect on nanofluid heat transfer.

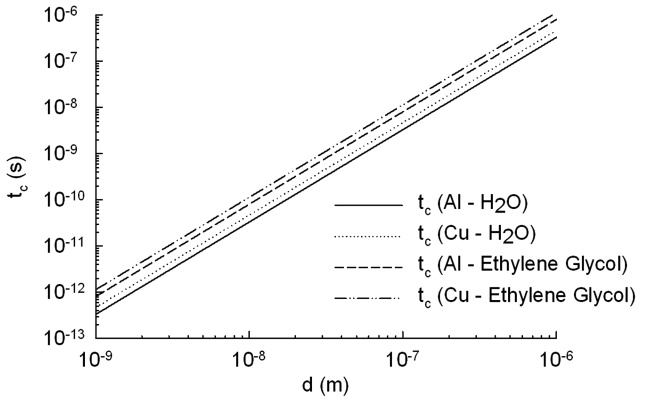

Fig. 1 t_c as a function of diameter for Cu and Al nanoparticles mixed in water and ethylene glycol.

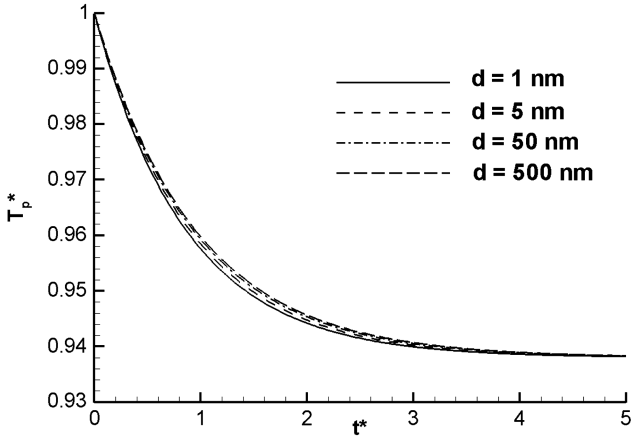

Fig. 2 Temperature as a function of time for $d_p = 1, 5, 50$, and 500 nm.

Figure 3 shows T_p^* as a function of t^* for aluminum nanoparticles in water, with a particle diameter of 50 nm, an initial temperature for the particles of 320 K, and initial temperature for the water of 300 K, for volume fractions of 0.0005, 0.005, and 0.05. These results show that the volume fraction is only significant if is greater than 1%. This corresponds to changing Π_2 while keeping Π_1 and Π_3 constant.

Figure 4 shows T_p^* as a function of t^* for aluminum nanoparticles in water, with a particle diameter of 50 nm, a volume fraction of 0.01,

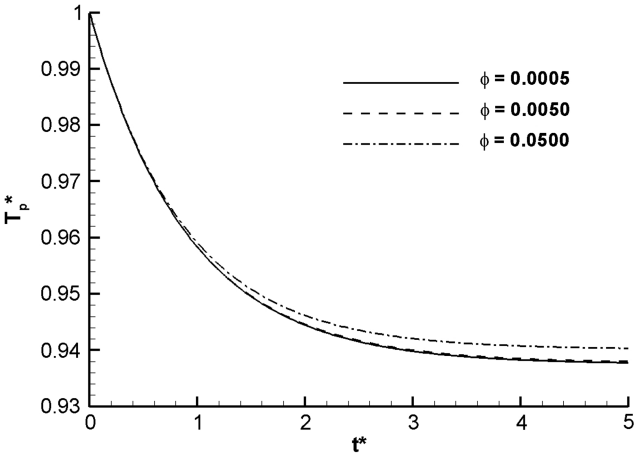

Fig. 3 Temperature as a function of time for $\varphi = 0.0005$, 0.005, and 0.05.

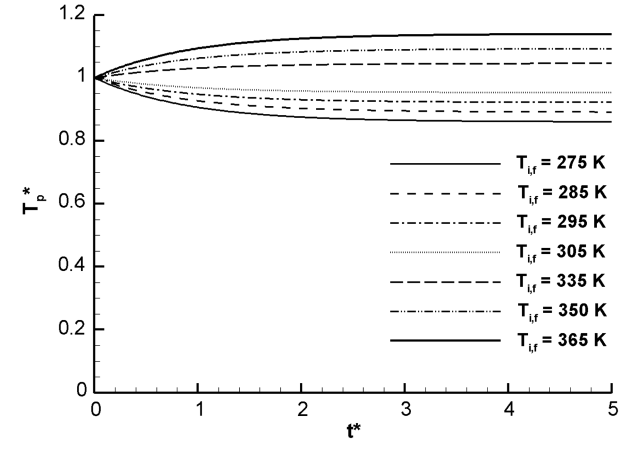

Fig. 4 Temperature as a function of time for $T_{f,i} = 275, 295, 305, 335, 350$, and 365 K.

Table 1 Comparison of theoretical vs computational results for α

Variables			*Nondimensional values*			*Exponents*	
Phi	$T_{f,i}$, K	D, nm	Π_1	Π_2	Π_3	Theory	Simulation
0.0100	320	1	0.00102	0.00592	0.9375	1.1883	1.1929
0.0100	320	10	0.00057	0.00592	0.9375	1.1085	1.1133
0.0100	320	100	0.00032	0.00592	0.9375	1.0636	1.0685
0.0100	320	1000	0.00018	0.00592	0.9375	1.0384	1.0433
0.0001	320	10	0.00057	0.000059	0.9375	1.1021	1.1067
0.0010	320	10	0.00057	0.00059	0.9375	1.1026	1.1073
0.0100	320	10	0.00057	0.00592	0.9375	1.1085	1.1133
0.0500	320	10	0.00057	0.03082	0.9375	1.136	1.1141
0.0100	270	10	0.00055	0.00592	1.11111	1.1042	1.1127
0.0100	280	10	0.00055	0.00592	1.07143	1.1051	1.1128
0.0100	290	10	0.00056	0.00592	1.03448	1.106	1.1129
0.0100	305	10	0.00057	0.00592	0.98361	1.1073	1.1131
0.0100	310	10	0.00057	0.00592	0.96774	1.1077	1.1132
0.0100	320	10	0.00057	0.00592	0.9375	1.1085	1.1133
0.0100	330	10	0.00058	0.00592	0.90909	1.1093	1.1134
0.0100	340	10	0.00058	0.00592	0.88235	1.1101	1.1135
0.0100	350	10	0.00059	0.00592	0.85714	1.1108	1.1136

an initial temperature for the particles of 320 K, and initial temperatures for the water ranging from 275 to 365 K. This corresponds to changing Π_3 while keeping Π_1 and Π_2 constant.

The results shown in Figs. 2–4, when combined with the time scale for t_c shown in Fig. 1, show that when the time scale for particles to reach equilibrium will range from the order of nanoseconds, for true nanoparticles, to microseconds, for particles with a diameter on the order of microns. This suggests that, with the possible exception of high-frequency systems, the particles are at the same temperature as the surrounding fluid. Current nanoparticle based heat transfer systems operate at time scales well above this limit [15].

IV. Analytic Approximation for Particle-Fluid Heat Transfer

Because these results appear to exponentially approach steady state, a general solution may be available. As t^* approaches infinity, and the system reaches thermal equilibrium, the limit will be given by

$$T_p^*(t^* \to \infty) = \left[\frac{(\Pi_2 + \Pi_3)}{(1 + \Pi_2)}\right] \tag{18}$$

The time-dependant temperature can be written as

$$T_p^*(t^*) = \left[\frac{(\Pi_2 + \Pi_3)}{(1 + \Pi_2)}\right] + \left[\frac{(1 - \Pi_3)}{(1 + \Pi_2)}\right] f(t^*) \tag{19}$$

Substituting this expression into (13) gives

$$f'(t^*) = (1 + \Pi_2)\left[1.0 + \Pi_1\left\{\left[\frac{(\Pi_2 + \Pi_3)}{(1 + \Pi_2)}\right] + \left[\frac{(1 - \Pi_3)}{(1 + \Pi_2)}\right] f(t^*)\right\}^{1/4}\right] f(t^*) \tag{20}$$

If $f(t^*)$ is assumed to be of order 1, then the differential equation can be solved analytically:

$$f(t^*) = e^{-\alpha t}, \qquad \alpha = (1 + \Pi_1)(1 + \Pi_2) \tag{21}$$

Table 1 compares these results to those obtained by curve-fitting to numeric solutions of the data. When the model is used to predict $f(t)$, the results are within 0.2% of the computational result for all of these cases. This suggests that the analytic model will be accurate enough to be integrated into a bulk heat transfer model.

V. Conclusions

Two conclusions can be drawn from this analysis. The first is that, for the majority of nanoparticle systems, the heat transfer between particles and the surrounding fluid occurs at an extremely rapid time scale. Because this means that the particles are at the bulk fluid temperature, it supports the hypothesis that Brownian motion drives the increase in thermal conductivity observed in these systems. The second conclusion is that, for cases where the time scales of the system require that transient heat transfer be computed, the proposed model fits to a degree of accuracy that is within the experimental error encountered in any thermal measurement.

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References

[1] Eastman, J. A., Choi, S. U. S., Li, S., Yu, W., and Thompson, L. J., "Anomalously Increased Effective Thermal Conductivities of Ethylene Glycol-Based Nanofluids Containing Copper Nanoparticles," *Applied Physics Letters*, Vol. 78, No. 6, 2001, pp. 718–720. doi:10.1063/1.1341218

[2] Das, S. K., Choi, S. U. S., Yu, W., and Pradeep, T., *Nanofluids: Science and Technology*, Wiley-Interscience, New York, 2008.

[3] Wang, X., Xu, X., and Choi, S. U. S., "Thermal Conductivity of Nanoparticle–Fluid Mixture," *Journal of Thermophysics and Heat Transfer*, Vol. 13, No. 4, 1999, pp. 474–480. doi:10.2514/2.6486

[4] Kumar, D. H., Patel, H. E., Kumar, V. R. R., Sundararajan, T., Pradeep, T., and Das, S. K., "Model for Heat Conduction in Nanofluids," *Physical Review Letters*, Vol. 93, No. 14, 2004, Article 144301.

[5] Lee, D., Kim, J.-W., and Kim, B. G., "A New Parameter to Control Heat Transport in Nanofluids: Surface Charge State of the Particle in Suspension," *Journal of Physical Chemistry B*, Vol. 110, No. 9, 2006, pp. 4323–4327. doi:10.1021/jp057225m

[6] Jang, S. P., and Choi, S. U. S., "Role of Brownian Motion in the Enhanced Thermal Conductivity of Nanofluids," *Applied Physics Letters*, Vol. 84, No. 21, 2004, pp. 4316–4318. doi:10.1063/1.1756684

[7] Prasher, R., Bhattacharya, P., and Phelan, P. E., "Thermal Conductivity of Nanoscale Colloidal Solutions (Nanofluids)," *Physical Review Letters*, Vol. 94, No. 2, 2005, Article 025901.

[8] Xuan, Y., Li, Q., Zhang, X., and Fujii, M., "Stochastic Thermal

Transport of Nanoparticle Suspensions," *Journal of Applied Physics*, Vol. 100, No. 4, 2006, Article 043507.

[9] Bahrami, M., Yovanovich, M. M., and Culham, J. R., "Assessment of Relevant Physical Phenomena Controlling Thermal Performance of Nanofluids," *Journal of Thermophysics and Heat Transfer*, Vol. 21, No. 4, 2007, pp. 673–680. doi:10.2514/1.28058

[10] Shukla, R. K., and Dhir, V. K., "Effect of Brownian Motion on Thermal Conductivity of Nanofluids," *Journal of Heat Transfer*, Vol. 130, No. 4, 2008, Article 042406.

[11] Nelson, I. C., Banerjee, D., and Ponnappan, R., "Flow Loop Experiments Using Polyalphaolefin Nanfluids," *Journal of Thermophysics and Heat Transfer*, Vol. 23, No. 4, 2009, pp. 752–761. doi:10.2514/1.31033

[12] Incropera, F. P., DeWitt, D. P., Bergman, T. L., and Lavine, A. S., *Fundamentals of Heat and Mass Transfer*, Wiley, Hoboken, NJ, 2006.

[13] Einstein, A., "Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen," *Annalen der Physik*, Vol. 322, No. 8, 1905, pp. 549–560. doi:10.1002/andp.19053220806

[14] Probstein, R. F., *Physicochemical Hydrodynamics*, Wiley-Interscience, New York, 2003.

[15] Ma, H. B., Wilson, C., Borgmeyer, B., Park, K., Yu, Q., Choi, S. U. S., and Tirumala, M., "Effect of Nanofluid on the Heat Transport Capability in an Oscillating Heat Pipe," *Applied Physics Letters*, Vol. 88, No. 14, 2006, Article 143116.